The study of number sense and teaching practice

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Abstract

The goal of this study was to investigate understanding of inservice elementary school teachers in Taiwan about number sense, teaching strategies of number sense and the development of number sense of students; and the profile of integrating number sense into mathematical instruction , and teaching practice. Data wase gathered through interviews of two elementary mathematics teachers regarding their understanding about number sense followed by observations of the teachers instructing in their mathematics classes. The data included the categorization and comparison of these teachers' understanding and teaching practices. The conclusions are as follows: The common point shared by two teachers was that in the teaching of four fundamental operations of fraction, they tended to ask the students to repeat and memorize the four fundamental operations of arithmetic or the arithmetic rules of addition, subtraction, multiplication and division of fraction. It was only the instruction valuing instrumental knowledge and could not develop the students' number sense.

Keywords: number sense, in-service teacher, teaching pratice, fractions, teacher education

INTRODUCTION

What is number sense? Number sense refers to a person's general understanding of numbers and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful and efficient strategies for managing numerical situations. (Reys & Yang, 1998; McIntosh al., 1999). The development of number sense is important in mathematics education. The National Council of Teachers of Mathematics, in their *Principles and Standards for School Mathematics*, note that number sense is one of the foundational ideas in mathematics in that students

> (1) Understand number, ways of representing numbers, relationships among numbers, and number system; (2) Understand meanings of operations and how they related to one another; (3) Compute fluently and make reasonable estimates. (NCTM, 2000, p32).

In many cases, however, much of the attention to developing number sense is a reaction to an over emphasis on computational procedures that are often algorithmic and devoid of number sense (Mclnotosh et al. , 1992., Yang, 1998). Over the past decades, the few studies that have investigated the mathematical understanding of elementary teachers indicate that they are not prepared to teach the mathematical subject matter entrusted to them (Cuff, 1993; Hungerford, 1994, Tsao, 2004, Tsao, 2005). The teachers play an important role in building number sense in the type of classroom environment they create, in the teaching practice they employ and in the activities they select (Tsao & Rung, 2007, 2008).

At present, the term “number sense” is not prevailing in Taiwan and most of the teachers have never encountered number sense, not to mention the attention on number sense instruction. However, the studies related to the teachers’ cognition of number sense and instructions are still insufficient; Yang (2000, 2002) has managed some studies upon the students’ capacities used when answering the questions of number sense and we still need further research on the teachers’ roles in number sense instruction. We not only have to allow the teachers to understand what number sense is, the importance of number sense, the spirit and practice of number sense involved in teaching, but also need to access to the students’ number sense performance. When helping the students to develop the number sense, what is the role of the teachers in number sense instruction? Do the teachers understand what number sense is? The result of this research will provide important data, allow the teachers to value the students’ number sense development and improve the students’ mathematics and problem-solving performance as the base for teacher educator to improve and strengthen on-the-job teachers’ mathematical knowledge or for teacher education institutions to design mathematics curriculum. At the time when Grade 1to 9 Curriculum gulidline proposed by Ministry of Education in Taiwan values number sense instruction, the exploration on teachers’ recognition on number sense and teaching practice becomes more significant. Therefore, this research used qualitative research method to explore elementary school teachers’ understanding toward the related knowledge of number sense and deeply accessed to the current situation in which the elementary school teachers integated number sense in teaching pratice.

REVIEW OF LITERATURE

The Commission on Standards for School Mathematics of NCTM, in 1987, described children with number sense as those children who understand number meaning, develop multiple relationships among numbers, know relative sizes of numbers, and comprehend how arithmetic operations affect results (Howden, 1989). The development of number sense is guided by a child's informal knowledge of numbers and quantity. Children need to be provided with problem solving opportunities that build on their own knowledge. By referring to how number sense was exhibited, Greeno (1991) characterized number sense in terms of flexible mental computation, numerical estimations and qualitative judgments. His perspective on number sense encompassed recognition of the role of equivalence in the decomposition/recomposition of numbers, the use of approximate numeric values in computational contexts and the making of inferences and judgments about quantities with numerical values. It seems intuitive that students who have more opportunities to learn and explore mathematics would develop greater number sense. The NCTM Curriculum and Evaluation Standards (1989) define that

> Children with good number sense (1) have well-understood number meaning, (2) have developed multiple relationships among numbers, (3) recognize the relative magnitude of numbers, (4) know the relative effect of operating on number, and (5) develop a referent for measures of common objects and situations in their environment (p. 38).

McIntosh et al. (1992) developed a number sense framework based on research and reflection on the literature related to the topic. Components of number sense hypothesized by several researchers (Sowder & Schapplle, 1989) were reviewed and analyzed, within the framework. Three broad categories emerged: knowledge of and facility with numbers, knowledge of and facility with operations, and ability to apply knowledge of and facility with numbers and operations to computational sittings.

Several researchers cited earlier made suggestions about how to facilitate students' development of number sense that will be presented in this section. First, the textbook, *Interactions* (Hope and Small, 1994), provides the following list of factors as an overview of general suggestions to facilitate number sense development:

> *Interactions* is based on the belief that children of all ages develop number sense in environments where they are encouraged to: (1).work with concrete materials and familiar ideas (2) discuss and share their discoveries and solutions (3) compose and recompose different arrangements and representations of numbers (4) investigate the realistic uses of numbers in their everyday world (5) explore number patterns and number relationships (6) create alternative methods of calculation and estimation (7)solve realistic problems using a variety of approaches(8) calculate for the purpose rather than for the sake of calculating (9) gather, organize, display, and interpret quantitative information (p.18)

Gurganus (2004) offers strategies for promoting number sense development across the grade levels. For example, measure and then make measurement estimates, plan powerful estimation experiences, explore very large number and representations, provide experience with number line, solve problem and consider the reasonableness of the solution, research number representation in other cultures, model the enjoyment number and number patterns. Number sense exhibits itself in a variety of ways as the learner engages in mathematical thinking. In essence, it is an important underlying theme as the learner chooses, develops and uses computational methods. The teacher is most critical factor in establishing a climate for curiosity and enjoyment of mathematics.

Sowder (1992) suggested that as teachers deal with the topic of number sense, they need to understand what characterizes number sense and need to prepare activities that present students with opportunities to explore within that framework. As students develop their "intuitive" feeling about number sense, teachers also need to know and recognize the dispositions that indicate the presence of number sense within the learner. Estimation and mental computation were two topics that are part of Sowder's conceptual framework that allowed learners to demonstrate an understanding of numbers and the structure of number systems. Thornton and Tucker (1989) suggested that teachers provide instruction which allows students to construct number meanings through realistic experiences. Particularly with the support of physical materials. They explained that in planning such lessons the teacher:

1. recognizes the importance of developing number sense
2. creates a positive climate for students to grow in their understanding and application of number
3. constructs situations that stimulate the development of number sense. (p.18)

Reys (1994) pointed out that teaching for the development of number sense requires conscious, coordinated effort to build connections and meaning on the part of the teacher. Teachers play an important role in building number sense in the type of classroom environment they create, in the teaching practices they employ and in the activities they select. Some strategies teachers might consider when teaching number sense are: (1)Use process questions. (2) Use writing assignments. (3) Encourage invented methods. (4) Use appropriate calculation tools. Number sense can be promoted by ensuring that students learn to calculate in various ways including written, mental, approximate, and electronic methods. (5) Help students establish benchmarks. Approximate computation or estimation is another important tool for encouraging students to use what they already know about numbers to make sense of new numerical situations. (6) Promote internal questioning. An important role for teachers in the development of number sense is helping students to learn to ask themselves key questions before, during and after the solution process. Teachers are a link in the chain of influence from reform to teaching and learning events. Furthermore, how the mathematics reform is implemented can be influenced by teachers' pedagogical content knowledge (Knapp, 1997).

Even & Tirosh(1995) point out that subject matter knowledge, knowledge about students' learning, as well as knowledge about mathematical instruction (Even & Tirosh, 1995; Shulman, 1986). The knowledge of what makes the learning of specific topics easy or difficult, and the method of teaching for understanding are vital aspects of teachers' cognition that relates to teachers' beliefs about pedagogical practice in the classroom (Swafford, Jones & Thornton, 1997). Most of these studies have been conducted within the interpretive tradition (Erickson, 1986), and have concentrated on providing rich descriptions of a small number of teachers in action in their classrooms; and inferences are drawn. These studies have made inferences between teachers' subject-matter knowledge and various aspects of the classroom. Experienced veteran teachers are usually compared to less experienced teachers; or a teacher is compared with himself in mathematical domains where he has more or less knowledge, such as the problem solving domain, the concepts domain, or the computational domain (Erickson, 1986).

Clearly, the way knowledge is organized and accessed as well as the nature of that knowledge is important. It must also be acknowledged that in many countries there has been a shift in focus from a transmission model of teaching to an emphasis on teaching for understanding (Fennema & Romberg, 1999). It is no longer a case of the student "working out what is in the teacher's head" but instead on teaching that aims to understand and build on what the student is thinking. This line of research is very important since here is where all aspects of teaching knowledge come together; and all must be considered to understand the whole.

METHODOLOGY

Research Design

This research adopted qualitative research method and since qualitative research valued in-depth and detailed exploration, the number of samples was usually limited (Bogden & Biklen, 1982). Upon this consideration, this research used small number of samples to proceed with in-depth interview and observation. At the first stage, the researcher managed semi-structural interview upon two participant teachers with respect to their understanding toward "the significance of number sense and children's number sense development" and to access to the participant teachers' views on number sense and children's number sense development. At the second stage, the researcher observed two teachers on-site teaching, accessed to the teachers' number sense integrating in teaching, and further analyzed the relation between their related knowledge of number sense and their teaching practice.

Interviews

Patton (1990) divided interviews into informal conversational interviews, general interview guide approach and standardized open-ended interview. This research adopted "informal conversational interviews" and "general interview guide approach". At the beginning of the interview, the researchers initially accessed to nine teachers' backgrounds, history of mathematics learning. Subsequently, the main interview content included two components. The first component was the teachers' recognition toward number sense. For example, have they ever heard of this term which is "number sense" ? How would they explain it? What is its importance? The second component referred to the teachers' cognition toward the children's number sense development. The interview content included: for the teachers, what kinds of mathematical abilities should the students possess? What kinds of characteristic do the students with number sense have? Each teacher received the interviews for three times and the interview time was about 90 minutes.

Observation on Natural Teaching Situation of Classroom

This research adopted the non-participant observation and the researchers did not participate in the activities observed and they only watched by "sitting on the boundary" (Fraenkel & Wallen, 2003). For understanding the participant teachers' teaching practice of number sense, the researchers proceeded with non-participant observation of "four fundamental operations of fraction" unit and proceeded with two-week non-participant observation of mathematics class with video and sound recording for data analysis. The researchers expected to find out the participants' most natural teaching behavior in the most realistic situations.

Backgrounds of Research Subjects

Teacher A had 16-year teaching experience and 6-year senior grade mathematics teaching experience in elementary school. When Teacher A studied at physical education department of teachers college, he still emphasized mathematics learning. The Teacher Braduated from language department of teachers college and is currently studying in institute of physical education in university of education. Teacher A has heard of the term "number sense" . Teacher A thought that during the mathematics teaching process, a teacher should understand the students' thinkings, needs and the capacities lacked at any time and then lead the students to have some interests for mathematics and increase the students' mathematics capacity in lives instead of resisting mathematics.

Teacher B graduated from art education department of teachers college. In her 11-year working experience, she had two-year administration experience and has been the art teacher for 3 years. For the recent 4 years, she has been the senior grade teacher. In recent years, she worked hard to cultivate herself and has been graduated from institute of visual art education. The teacher thus had special talents in art and human aspects. Teacher B was devoted to the students and expected the students to have diverse skills. Since she taught mathematics in recent years, she had close connection with mathematics. Teacher B thought she needed mathematics, thus, she now has close relationship with mathematics. Thus, "number sense" was strange for Teacher B. We could also understand from the interview for Teacher B with respect to students' number sense development that Teacher B's knowledge of number sense was insufficient.

Data Analysis

In qualitative research, data analysis was mainly to combine the data acquired by the researcher from various sources, such as observation, interview and content analysis. This research used sound recording to record the interviews and after translating the interview of "the significance of number sense, general situation of number sense combined in teaching and children's number sense ability development" into the scripts, the researcher repetitively read the scripts and analyzed the participant teachers' responses to clarify the participant teachers' related knowledge of number sense. As to non-participant observation, in order to understand the participants' teaching behavior, the researchers used video and sound recording to have the non-participant observation in the classroom and translated the on-site teaching into scripts for data analysis.

RESULTS AND DISCUSSIONS

Teachers' Understanding toward the Significance of number sense and Importance

Teacher A had deeper study on number sense. Teacher A indicated that number sense was "a kind of instinct toward numbers and it was developed gradually." Number sense was built from the children's daily life experience since their childhood. By the cultural stimulation of number concepts at home, the children would develop their own understanding toward number pattern and could further judge the amount of numbers and understand the influence of arithmetic on numbers. After entering the school, the conceptual knowledge of numbers would develop with the related process knowledge such as number counting skills, the identification and writing of numbers. The children could master the understanding and application of the number concept, think flexibly and have logic reasoning; thus, their number sense would be more complete.

Teacher A indicated that number sense should be established since childhood and be constructed by gradual development; if the children's number sense was not established well in childhood, it would affect his mathematics learning in the future.

Teacher A: …in fact, the key relies on his future learning such as his arrangement of numbers, size of space and the change of some numbers. It is actually significant to some degree!

Teacher A pointed out that in childhood, number sense has started to be developed; in daily lives, parents counted the fingers and toys for their children. The pre-schoolers would gradually have some ideas of numbers and counting numbers. The establishment of the initial number sense would further affect their arithmetic learning in the future. The logic thinking of numbers, arrangement of numbers and the model of problem-solving strategies would make the children to have more feelings toward numbers with their growing. They would have some basic concepts instinctively for the numbers they saw and heard.

Teacher B never has heard of "number sense" this term. Afetr researchers explain meaning of number sense. Teacher B defined the number sense in narrow sense that it was the understanding toward the significance of numbers and dealing with numbers as quantities instead of treating numbers as the instinct and perception of abstract and formal things. Teacher B thought that in daily lives, the application of number sense or mathematics was not relatively important.

Teacher B: …actually number sense is the sensitivity to numbers. He might acquire it through constant practice and generalize in his mind. Sometimes the teachers have to remind them of this, such as ratio of the circumference, 3.14….

In current teaching, the teachers tended to value the precise answers and mathematical teaching at school also spent most of the time to make the students have the mechanical practice of calculation which resulted in the situation in which the children's inherent instinct and flexible thinking of mathematics aspect were gradually limited to rapid and effective arithmetic rules and formulas. Thus, the teachers' in-depth understanding toward number sense, spending more time to maintain and increase the children's number sense capacity and flexibly using proper, creative and efficient strategies to solve the problems will increase the students' interests of mathematical learning.

Teachers' Cognition toward the Students' Number Sense

Teacher A and B proposed that the children with number sense could think more flexibly and have logic reasoning.

Teacher B: …For example, recently we are solving a question: there is a circle and a diameter. How many sectors are there? There are two diameters. How many sectors are there? There are three diameters. How many sectors are there?…some children realized it immediately! Two times. It becomes two after one cut and four after two cuts. Thus, it is 2x. Some children are quick and I think it is number sense!

Teacher A thought that the students who could understand the significance of numbers and their relations, develop different problem-solving strategies and have reasonable judgment possessed good number sense ability. Teacher A pointed out that the children with number sense ability could distinguish the amount of numbers and indicated that the children's identification of the influence of arithmetic on numbers was the indicator of number sense performance.

Teacher B thought that the children who could manage the abstract thinking of mathematics had better number sense.

Teacher B: First, he was more interested in mathematics and did not resist numbers. Secondly, he might have stronger abstract thinking and more imagination. Thirdly, he might have stronger ability to memorize, or even he had stronger calculation ability so that he could influence these numbers immediately.

Number Sense integrated into Teaching Practice

The researcher observed the lesson of "four fundamental operations of fraction" in the classroom in order to understand the participant teachers' number sense integrated into teaching practice, the researchers analyzed the relationship between the teachers' number sense integrated into teaching practice and the teachers' related knowledge of number sense.

Teacher A's Number Sense Integrated in Teaching Practice

Teacher A was a "narrator" in teaching. After arranging the questions, the teacher was good at "questioning the students" and stressed the understanding toward the significance of mathematics. The teacher also emphasized "procedure knowledge" instead of "conceptual knowledge". Teacher A considerably emphasized the principle of arithmetic. When solving the questions, he always asked the students to read the "pithy formula" again. Thus, three minutes before the class, Teacher A usually asked the students to read the pithy formula of four fundamental operations of integer again. At the beginning of the unit "four fundamental operations of fraction", Teacher A first reviewed the students' four fundamental operations of arithmetic in integer and addition, subtraction, multiplication and division of fraction. At the grade fourth, the children has already been familiar with four fundamental operations of arithmetic in integer. Thus, Teacher A asked that "what is the game rule of four fundamental operations of arithmetic in integer?" The students answered immediately "addition and subtraction after multiplication and division." Teacher A proposed the calculation right away to strengthen the description: For example:

Teacher A question arrangement: 3+3×2

Teacher A: How many kinds of arithmetic are there in this equation ?

Student: Two kinds.

Teacher A: Which are?

Student: Addition and multiplication.

Teacher A: Let us recall, what are the rules of four fundamental operations of arithmetic?

Student: Addition and subtraction after multiplication and division.

Question arrangement: after explaining four fundamental operations of arithmetic in integer, Teacher A arranged the question and asked one students to present the problem-solving process on the blackboard. After the student finished, the teacher asked the whole class to judge if the said student's problem-solving was correct. All students in class could directly judge the correctness of the answer. Thus, Teacher A asked the students again:

Teacher A: What does four fundamental operations of arithmetic mean ?

Student: Addition, subtraction, multiplication and division.

Teacher A: How to manage the arithmetic ?

After the students generally understood and reviewed the four fundamental operations of arithmetic in integer, Teacher A asked them verbally to review the fractions.

Teacher A: What are the types of fractions? How about the one less than 1, what does it call?

Student: Proper fraction.

Teacher A: in four fundamental operations of arithmetic, we have addition and subtraction, then, when we manage addition and subtraction of fractions with different denominators, what do you do?

Teacher A: Looking for the lowest common multiple of denominators. What is the step I mentioned before?

Student: Reduction to a common denominator.

Teacher A: For multiplication and division of fractions, what is the first step?

Student: Addition and subtraction after multiplication and division.

Teacher A: I mean multiplication and division of the fractions? Think more.

Student: Divisor numerator and denominator become reciprocal.

The teacher started the major lesson. First, Teacher A asked the whole class to read it again. Then the teacher started to describe the meaning of the question. The reason that many students failed to solve the writing questions was that they did not understand the questions. Thus, Teacher A listed the related situation questions in daily lives, introduced integer, pointed out some key points in the question by questioning to show the clues or concepts in the question and guide the students to manage successful equation. The teacher also introduced the meanings of each number in the equation, the significance of each arithmetic, and the meaning of arithmetic results. Teacher A posed the questions to further clarify the students' concept. When the mathematical question presented by the symbols matched the learners' concept, the learners could give the symbols meanings according to the concepts. When the learners had multiple meanings for one symbol, they could have constant understanding (Fennema, 1933). In order to allow the students to understand the meaning of the questions, Teacher A used diagrams or re-arrangements to combine the situations with daily lives in verbal method. For example:

Original question: Mother bought $4\frac{1}{2}$ kilogram of flour and used $1\frac{3}{8}$ kilogram for making cookies and $\frac{9}{4}$ kilogram for making bread, how many kilograms of flour left?

Re-arrangement: If your mother gives you \$100 and you use \$30 for buying pencils and \$20 for buying drinks, how much do you leave?

Teacher A: Question like this, how do you ponder on this question?

Students: $100-30-20$

Teacher A: That's arithmetic, I ask you how to think?

Teacher A: Reducing what is used at the first time and the second time, we will have what's left. So, let's come back, how many kilograms of flour did we have?

Teacher A: What does $4\frac{1}{2}-1\frac{3}{8}$ mean ?

Teacher A: Besides this equation, do we have other equations? What I said about $100-30-20$……

Teacher A: Any other arithmetic methods?

Students: Adding 30 and 20 first.

Teacher A: that means to add the money you spent.

Teacher A: What do you find ? ……although the methods are different, the answers are the same. Return to this question, how do your arrange it?

Students: $4\frac{1}{2}-\ (1\frac{3}{8}+\frac{9}{4})$

Teacher A: What does $(1\frac{3}{8}+\frac{9}{4})$ mean?

Students: the flour used

After Teacher A arranged the equation, he would keep on repeating the descriptions and expect the students to understand the meanings of numbers and arithmetic in the equation after re-arrangement. When managing the problem-solving, Teacher A repeatedly asked the students to answer, what do we do for this kind of question. For example, when having the addition and subtraction of fraction, Teacher A asked that "when we manage addition and subtraction of fractions with different denominators, what do we do first?" "Students: "reduction to a common denominator" Teacher A: "for reducing to a common denominator, we have to find…?the lowest…" Students: "lowest common multiple."

When finishing teaching one question pattern of concept, depending on the time of the course, Teacher A would decide if the students could have independent practice. After the students finished, Teacher A individually checked the students' various solutions. According to the students' problem-solving patterns or wrong problem-solving, the teacher asked the students to present in front and the whole class examined if the problem-solving process was reasonable and the calculation was correct. However, when most of the students make the same mistake, Teacher A would pose the question and solve it again. Teacher A often asked the student, "do you have other solutions ", to encourage the students the construct different problem-solving strategies. After class, the teacher would give the students re-arranged questions for further practice at home.

Comparing the Relation Between Teacher A's Teaching Pratice and Teacher's Related Knowledge of Number Sense

We observed Teacher A's "four fundamental operations of fraction" teaching and focused on Teacher A's proposal that the upgrading of number sense could be based on the construction and extension of basic concepts. The researcher found out that Teacher A considerably valued the connection between old and new concept. Thus, Teacher A had the review of lead concepts. Although Teacher A emphasized the construction of concepts, according to his teaching, we found out that Teacher A focused on memorizing principles in mathematics classroom and mathematics formulas. For example: when managing addition and subtraction of fractions, he tended to repeat" first we should manage the reduction to a common denominator", "the rule of four fundamental operations of arithmetic was addition and subtraction after multiplication and division….". With regard to the emphasis of arithmetic rules and not leading the students to understand the influence of arithmetic on numbers, many studies have proved that this kind of formula memorizing and arithmetic were meaningless for the students' mathematics learning and helpless of the students' number sense development (Yang, 2000; Markovits & Sodwer, 1994) .

Teacher A thought that the classroom discussion construction could increase the number sense. However, in the unit "four fundamental operations of fraction", Teacher A's clarification on the meaning of concept of the questions were based on narration or question and did not give the students the opportunity of group and class discussion. Although Teacher A emphasized the influence of process question on number sense development, in the teaching of four fundamental operations of fraction, Teacher A thought that the key of the unit was the skill of four fundamental operations of arithmetic. Teacher A guided the students' thinking by questioning, including the next step of problem-solving, the key sentence of the writing question or the meanings of numbers and arithmetic. The teacher expected to increase the students' understanding toward mathematics through the students' answers.

Fractions were the stumbling stones for the children's mathematics learning, particularly the application questions. The students originally had difficulty with the understanding of the meaning of writing, with fractions which were like "unknown book", the students had more obstacles on the understanding toward mathematics writing questions. In order to allow the students to understand the meanings of the questions and solve the problems of fraction figures, Teacher A would usually change the questions slightly, combine the students' daily life experience and change the figures into "integers" which were more familiar to the students in order to lead the students to correctly construct problem-solving strategies.

Although Teacher A proposed that the development of the students' standard point construction could facilitate number sense ability, Rey (1994) pointed out that only encouraging the students' application of reference point could judge the rationality of the answers and facilitate the understanding of fraction concepts. However, even when encountering the comparison problem in fractions, Teacher A still tended to ask the students to calculate the answers one after another. Thus, Teacher A's teaching lacked of the point of establishing the students' reference point ability .

Teacher A thought that practicing could increase the students' number sense ability. From the teaching of four fundamental operations of fraction, Teacher A still arranged the questions everyday and asked the students to practice at home. The teacher could observe the students' practice in the classroom and the students would practice more carefully and establish their confidence of problem-solving (Thornton & Tucker, 1989). However, when in class, because of the limitation of time, Teacher A rarely allow the students to have individual practice in the classroom.

Teacher A thought that the operation of concrete objects, the involvement of lines and diagrams in mathematics teaching would increase the students' number sense. For example, in four fundamental operations of fraction, there were questions related to speed. Although the teacher has taught the speed issue in the last unit, the students were still unclear about the concept of "relative speed". In order to clarify the students' concepts, Teacher A would ask the students with vague concepts to actually experience the relative speed and describe with lines.

Teacher B's Number Sense Integrated in Teaching

Teacher B was a "presenter" and "problem-solving person" in teaching; when teaching the problem-solving methods in the textbooks. She tended to directly tell the students the problem-solving methods or ask the students to directly read the problem-solving methods listed in the group discussion of textbook and explore the meanings of numbers or arithmetic in the equation and further presented them verbally in group. When explaining the problem-solving process, Teacher B would properly ask the whole class or the reporters and expected to lead to the teacher's discussion through this kind of question and examine if the students really understand; at the beginning of the class, Teacher B did not review. First, the teacher asked the students to read the questions in the textbook and divided the class into four groups. The students in each group discussed the meaning and finished the problem-solving process according to the problem-solving strategies described in the textbook. After the discussion, the students presented the problem-solving process on the blackboard and the students of each group selected on representative to explain the problem-solving process and the meanings of each numbers and arithmetic. During the process of the students' work, the teacher properly questioned them and clarified the meanings of each number and arithmetic and to understand if the students' problem-solving cognition was proper. However, the teacher only emphasized the process and rules of four fundamental operations of arithmetic.

For example: students' problem-solving method was below:

$$4\frac{1}{2}-1\frac{3}{8}-\frac{9}{4}=\frac{9}{2}-\frac{11}{8}-\frac{9}{4}=\ldots\ldots$$

Teacher B further described that: "the calculation was good. First of all, you have to transform all fractions into improper fractions, ……remember to use continuous subtraction, from which part to which part ?" The students raised their hands immediately: "from left to right." Form this point, we can also find out that Teacher B's mathematical concept involved some conflict. The subtraction of fractions did not have to be all transformed into improper fraction for subtraction arithmetic. In other words, it was not an arithmetic rule. Thus, Teacher B's mathematics knowledge would be the critical factor influencing mathematics teaching.

With regard to the students' wrogn problem-solving, Teacher B spent less time to calculate the devleopmetn of arithmetic and immediately told the students the arithmetic process and did not give the students time to think of problem-solving strategies. The frequent application of problem-solving diagrams would increase the students' understanding toward the meanings of writing question which resulted in successful problem-solving.
One of the group of students pointed out the error of another group's problem-solving. The method was below:

$$1\frac{3}{8}+\frac{9}{4}=1\frac{3}{8}+2\frac{1}{4}=\ldots\ldots$$

Teacher B immediately told the students that the method was wrong: "the addition was wrong and you could not equal them like this! " Teacher B then asked the students to correct immediately. The students added [1] on the right equal sign of $\frac{3}{8}$. After the students finished the description, Teacher B added that if they respectively calculated the integers: "integer was integer and fraction was fraction. " in the following equation, the equal sign was established.

$$1\frac{3}{8}+\frac{9}{4}=1\frac{3}{8}+2\frac{1}{4}=\ (1+2)\ +\ (\frac{3}{8}+\frac{1}{4})$$

The teacher also further describe the significance of "=" in mathematical symbols and reminded the students not to make the same mistake again..

After four groups of students finished the report, Teacher B did not make the students to actively find out the relations among these four problem-solving strategies with different perceptions; instead, she directly told the students that solution 1 and 3 as indicated in Table 1 (Appendix) were based on the same perception; solution 2 and 4 as indicated in Table 1 (Appendix) were based on the same perception strategies. After finishing the problem-solving of the first question, Teacher B started to remind the students' memory and asked the students to present the rules of four fundamental operations of arithmetic and wrote them on the blackboard, such as: (1) addition and subtraction after multiplication and division; (2) arithmetic first in brackets ; (3)When there were no brackets, calculating from left to right. Teacher B first discussed the rules of these calculations. After finishing one page, the teacher asked the students to read "the suggestion for teachers" in the textbook and underline the key points. There were the practices for calculation and writing questions in each question pattern. Teacher B asked the students to practice independently and asked the first student finished problem-solving to write down the problem-solving process on the blackboard. During the students' problem-solving process, Teacher B constantly reminded the student of the arithmetic rule. When questioning, the teacher only emphasized the process and rules of four fundamental operations of arithmetic. After all students finished the problem-solving, the researcher managed the review.

When starting the second question pattern, likewise, Teacher B asked the students to read the questions aloud and directly listed the equation on the blackboard. The teacher directly explained the problem-solving skills. For example, the division of fraction was to multiply the reciprocal of divisor. After Teacher B finished the first problem-solving strategy, he (she) then asked, "is there any faster method? Connecting the equation?" The students immediately proposed the second equation:

$$324 \div 4\frac{1}{2} \div 1\frac{5}{7} =$$

Teacher B: We do not first transform it into reciprocal, what should we do first?

Student: Fraction reduction! ,transforming it into improper fraction first !

Teacher B: Remember to write this equation, it won't be wrong!

Teacher B: Read the following point again!

Teacher B: Besides this equation, we can also write: $324 \div (4\frac{1}{2} \times 1\frac{5}{7})$, but, you have to count what's in the brackets first! Understand?

Teacher B:In this unit, it is easy to make mistakes ! Please calculate step by step, you can transform them into improper fraction, or……, don't be lazy! As long as you have one step wrong, the whole process would be wrong!

The calculation itself was problem-solving. Teacher B treated calculation as problem-solving for teaching. However, during the teaching process, Teacher B only described verbally and calculated and the students did not participate actively. We did not know if the students really understand the method and meaning of arithmetic. Thus, in the mathematics classroom, the students' participation was not active as expected.

After finishing the explanation in the textbook, Teacher B reviewed some patterns in the practice books. According to the "re-arrangement" in the practice books, Teacher B found out the related questions and explained them again. For example, Teacher B also asked the students to read the questions again and wrote down the key sentences and figures on the blackboard. Then the teacher listed the equation and further asked the students to read the key points in the practice books:

Teacher B: Good! The formula of this question is:

$$200 \div (36 \div \frac{2}{3}) \ldots\ldots$$

please read the key point 3 on page 29!

Student: when managing the continuous division of fractions, first transforming the mixed fractions in the equation into improper fraction and reversing the numerator and denominator of divisor. According to the multiplication of fractions, we can have the answer.

Teacher B: Good ! this one is……

With regard to more complicated questions, Teacher B used lines, diagrams to present the meaning on the blackboard and explain the questions! Usually Teacher B would directly present the equation and ask the students directly: "Understand? Understand? "Or directly tell the students the problem-solving skills: "……if your are not clear about the question, you set up certain number directly and solve it! ……when there are unknown numbers, you set them up first ……" . Thus, Teacher B could obviously adopt "teacher's presentation---student's imitation" . When facing the mathematical problems, the teacher was always eager to give the students the most efficient and correct solutions. Although she only questioned in the equation writing process, the teacher rarely provided the students the chances to actively construct problem-solving strategies. The teaching only focusing on instrumental knowledge would result in the students' neglect of the connections among the concepts.

Comparing the Relation Between Teacher B's Teaching Pratice and Teacher's Related Knowledge of Number Sense

Teacher B thought that class discussion could increase the students' number sense ability. In the teaching of the whole unit, there was only one question pattern upon discussion. The teacher expected, through group discussion, the students could understand the meaning of each number and arithmetic in the formula and finish problem-solving. Therefore, we should pay attention to the point if Teacher B efficiently created a discussion environment in the mathematics teaching. Teacher B thought that practice was the only way to increase number sense ability. Thus, besides allowing the students to practice individually in the classroom, Teacher B provided new concept questions every day for practices. Teacher B expected the repetitive practice could allow the students to master arithmetic rules and further upgrade the students' sensitivity to fraction and fraction arithmetic.

Teacher B also proposed that the construction of the basic mathematics concept was the base of number sense. However, from Teacher B's problem-solving teaching, we find out that: after arranging the questions, Teacher B only simply described the meaning of the question and did not alow the students to think and construct their own problem-solving strategies. The whole class was only Teacher B's own question arranging and problem-solving. The class was boring. Thus, we find out that the participation of Teacher B's students was not enthusiastic and the learning effect was relatively reduced.

CONCLUSIONS AND RECOMMENDATIONS

Teacher A's teaching pratice of four fundamental operations of fraction unit, including the construction of basic concept, combining mathematics in daily lives, using process questions, efficiently using concrete objectives, lines, graphs and practices. However, Teacher A neglected group and class discussion and reference point cultivation in the mathematical teaching. For Teacher B, besides allowing the students to practice which met her number sense combined in teaching strategy, class discussion, the construction of basic mathematics concepts were significantly neglected in her four fundamental operations of fraction instruction.

The common point shared by two teachers was that in the teaching of four fundamental operations of fraction, they tended to ask the students to repeat and memorize the four fundamental operations of arithmetic or the arithmetic rules of addition, subtraction, multiplication and division of fraction. It was only the instruction valuing instrumental knowledge and could not develop the students' number sense (Yang, 1999). Grade 1 to 9 Curriculum emphasized meaningful learning. However, after analyzing two participant teachers, we found out that there were tended to use traditional formulas for teaching. Many studies (Markovits & Sowder, 1994; Yang, 2000; Reys & Yang, 1998; Yang, 2002) also proved that when the students solved the problems of numbers and memorized a series of formulas, they only operated the standard calculation and did not really understand the numbers. In other words, they were lack of number sense with teaching pratcie.

The teachers with mathematical and physical backgrounds such as Teacher A graduating from Physical Education Department obviously had more abundant related knowledge of number sense; the teachers without mathematical and physical backgrounds and were not interested in mathematics such as Teacher B graduating from visual art department had less related knowledge of number sense. Both of the teachers indicated in the interview that during their teacher education process, they did not encount the knowledge related to number sense and the mathematics teaching method they learned was traditional narration.

"Number sense" has been valued by many advanced countries. However, in mathematics education field, it was particularly strange for the teachers at the first teaching site. Most of the teachers have never encountered number sense, not to mention their value for number sense intergated in teaching. The teachers must first improve their number sense and related knowledge of number sense. A teacher with number sense and related knowledge of number sense can stimulate and guide the students to increase number sense. Therefore, the educational administration should hold the related studies for the basic teachers, enhance their understanding toward number sense so that number sense can be efficiently integrated into mathematics teaching for proceeding with meaningful mathematics teaching, and develop the students' mathematical capacities.

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APPENDIX

Table 1.

Solution 1: $4\frac{1}{2}-1\frac{3}{8}=3\frac{1}{8}$ $3\frac{1}{8}-\frac{9}{4}=\ldots\ldots$	Solution 2: $1\frac{3}{8}+\frac{9}{4}=3\frac{5}{8}$ $4\frac{1}{2}-3\frac{5}{8}=\ldots\ldots$
Solution 3: $4\frac{1}{2}-1\frac{3}{8}-\frac{9}{4}=\ldots\ldots$	Solution 4: $4\frac{1}{2}-\left(1\frac{3}{8}+\frac{9}{4}\right)=\ldots\ldots$